# 4.6 Combined\_mutually exclusive\_conditional independence\_prob diagram\_P\_2

**1a.** *[3 marks]*

A café serves sandwiches and cakes. Each customer will choose one of the following three options; buy only a sandwich, buy only a cake or buy both a sandwich and a cake.

The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45.

Find the probability that a customer chosen at random will buy

both a sandwich and a cake.

**1b.** *[1 mark]*

only a sandwich.

**1c.** *[1 mark]*

On a typical day 200 customers come to the café.

Find the expected number of cakes sold on a typical day.

**1d.** *[3 marks]*

Find the probability that more than 100 cakes will be sold on a typical day.

**1e.** *[1 mark]*

It is known that 46 % of the customers who come to the café are male, and that 80 % of these buy a sandwich.

A customer is selected at random. Find the probability that the customer is male and buys a sandwich.

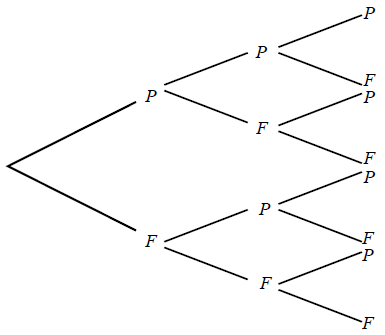
**1f.** *[4 marks]*

A female customer is selected at random. Find the probability that she buys a sandwich.

**2a.** *[3 marks]*

Iqbal attempts three practice papers in mathematics. The probability that he passes the first paper is 0.6. Whenever he gains a pass in a paper, his confidence increases so that the probability of him passing the next paper increases by 0.1. Whenever he fails a paper the probability of him passing the next paper is 0.6.

Complete the given probability tree diagram for Iqbal’s three attempts, labelling each branch with the correct probability.



**2b.** *[2 marks]*

Calculate the probability that Iqbal passes at least two of the papers he attempts.

**2c.** *[3 marks]*

Find the probability that Iqbal passes his third paper, given that he passed only one previous paper.

**3a.** *[2 marks]*

Willow finds that she receives approximately 70 emails per working day.

She decides to model the number of emails received per working day using the random variable , where  follows a Poisson distribution with mean 70.

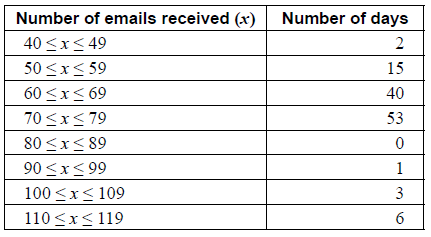
Using this distribution model, find .

**3b.** *[2 marks]*

Using this distribution model, find the standard deviation of .

**3c.** *[3 marks]*

In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.



From the table, calculate

an estimate for the mean number of emails received per working day.

**3d.** *[2 marks]*

an estimate for the standard deviation of the number of emails received per working day.

**3e.** *[1 mark]*

Give one piece of evidence that suggests Willow’s Poisson distribution model is not a good fit.

**3f.** *[3 marks]*

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of  emails per day.

Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of *λ*.

**3g.** *[5 marks]*

Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of *λ*.

**4.** *[5 marks]*

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

**5.** *[7 marks]*

Each of the 25 students in a class are asked how many pets they own. Two students own three pets and no students own more than three pets. The mean and standard deviation of the number of pets owned by students in the class are  and  respectively.

Find the number of students in the class who do not own a pet.

**6a.** *[2 marks]*

Events  and  are such that  and .

Find .

**6b.** *[2 marks]*

Find .

**6c.** *[2 marks]*

Hence show that events  and  are independent.

**7a.** *[3 marks]*

Consider two events  and  such that  and .

Calculate ;

**7b.** *[3 marks]*

Find .

**8a.** *[2 marks]*

There are 75 players in a golf club who take part in a golf tournament. The scores obtained on the 18th hole are as shown in the following table.



One of the players is chosen at random. Find the probability that this player’s score was 5 or more.

**8b.** *[2 marks]*

Calculate the mean score.

**9a.** *[3 marks]*

John likes to go sailing every day in July. To help him make a decision on whether it is safe to go sailing he classifies each day in July as windy or calm. Given that a day in July is calm, the probability that the next day is calm is 0.9. Given that a day in July is windy, the probability that the next day is calm is 0.3. The weather forecast for the 1st July predicts that the probability that it will be calm is 0.8.

Draw a tree diagram to represent this information for the first three days of July.

**9b.** *[2 marks]*

Find the probability that the 3rd July is calm.

**9c.** *[4 marks]*

Find the probability that the 1st July was calm given that the 3rd July is windy.

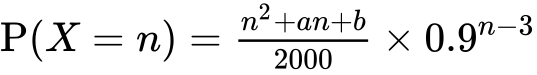
**10a.** *[3 marks]*

A Chocolate Shop advertises free gifts to customers that collect three vouchers. The vouchers are placed at random into 10% of all chocolate bars sold at this shop. Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let  be the probability that Kati obtains her third voucher on the  bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

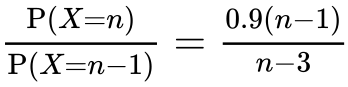
Show that  and .

**10b.** *[5 marks]*

It is given that  for .

Find the values of the constants  and .

**10c.** *[4 marks]*

Deduce that  for .

**10d.** *[5 marks]*

(i)     Hence show that  has two modes  and .

(ii)     State the values of  and .

**10e.** *[3 marks]*

Kati’s mother goes to the shop and buys  chocolate bars. She takes the bars home for Kati to open.

Determine the minimum value of  such that the probability Kati receives at least one free gift is greater than 0.5.

**11a.** *[2 marks]*

Six balls numbered 1, 2, 2, 3, 3, 3 are placed in a bag. Balls are taken one at a time from the bag at random and the number noted. Throughout the question a ball is always replaced before the next ball is taken.

A single ball is taken from the bag. Let  denote the value shown on the ball.

Find .

**11b.** *[3 marks]*

Three balls are taken from the bag. Find the probability that

the total of the three numbers is 5;

**11c.** *[3 marks]*

the median of the three numbers is 1.

**11d.** *[3 marks]*

Ten balls are taken from the bag. Find the probability that less than four of the balls are numbered 2.

**11e.** *[3 marks]*

Find the least number of balls that must be taken from the bag for the probability of taking out at least one ball numbered 2 to be greater than 0.95.

**11f.** *[8 marks]*

Another bag also contains balls numbered 1 , 2 or 3.

Eight balls are to be taken from this bag at random. It is calculated that the expected number of balls numbered 1 is 4.8 , and the variance of the number of balls numbered 2 is 1.5.

Find the least possible number of balls numbered 3 in this bag.

**12a.** *[2 marks]*

The events  and  are such that ,  and .

Find .

**12b.** *[2 marks]*

Hence show that the events  and  are independent.

**13a.** *[6 marks]*

A survey is conducted in a large office building. It is found that  of the office workers weigh less than  kg and that  of the office workers weigh more than  kg.

The weights of the office workers may be modelled by a normal distribution with mean  and standard deviation .

(i)     Determine two simultaneous linear equations satisfied by  and .

(ii)     Find the values of  and .

**13b.** *[1 mark]*

Find the probability that an office worker weighs more than  kg.

**13c.** *[2 marks]*

There are elevators in the office building that take the office workers to their offices.

Given that there are  workers in a particular elevator,

find the probability that at least four of the workers weigh more than  kg.

**13d.** *[3 marks]*

Given that there are  workers in an elevator and at least one weighs more than  kg,

find the probability that there are fewer than four workers exceeding  kg.

**13e.** *[3 marks]*

The arrival of the elevators at the ground floor between  and  can be modelled by a Poisson distribution. Elevators arrive on average every  seconds.

Find the probability that in any half hour period between  and  more than  elevators arrive at the ground floor.

**13f.** *[3 marks]*

An elevator can take a maximum of  workers. Given that  workers arrive in a half hour period independently of each other,

find the probability that there are sufficient elevators to take them to their offices.

**14.** *[6 marks]*

Josie has three ways of getting to school.  of the time she travels by car,  of the time she rides her bicycle and  of the time she walks.

When travelling by car, Josie is late  of the time. When riding her bicycle she is late  of the time. When walking she is late  of the time. Given that she was on time, find the probability that she rides her bicycle.

**15a.** *[2 marks]*

The continuous random variable  has the probability distribution function .

Find the value of  to three decimal places.

**15b.** *[2 marks]*

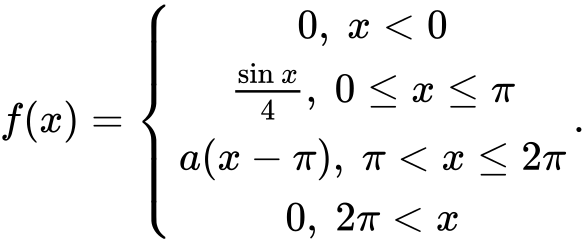
Find the mode of .

**15c.** *[2 marks]*

Find the value .

**16a.** *[2 marks]*

The probability density function of a continuous random variable  is given by

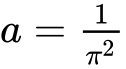


Sketch the graph .

**16b.** *[2 marks]*

Find .

**16c.** *[3 marks]*

Show that .

**16d.** *[1 mark]*

Write down the median of .

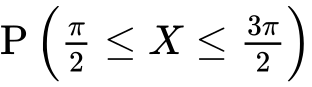
**16e.** *[3 marks]*

Calculate the mean of .

**16f.** *[3 marks]*

Calculate the variance of .

**16g.** *[2 marks]*

Find .

**16h.** *[4 marks]*

Given that  find the probability that .

**17a.** *[3 marks]*

The random variable  follows a Poisson distribution with mean .

Given that , show that .

**17b.** *[4 marks]*

Given that , find the probability that .

**18a.** *[6 marks]*

Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of  and standard deviation of .

(i)     Calculate the percentage of Suzie’s turnips that weigh between  and .

(ii)     Suzie has  turnips to take to market. Find the expected number weighing more than .

(iii)     Find the probability that at least  of the  turnips weigh more than .

**18b.** *[6 marks]*

Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of . Ray only takes to market turnips that weigh more than . Over a period of time, Ray finds he has to reject  in  turnips due to their being underweight.

(i)     Find the standard deviation of the weights of Ray’s turnips.

(ii)     Ray has  turnips to take to market. Find the expected number weighing more than .

**19a.** *[3 marks]*

Natasha lives in Chicago and has relatives in Nashville and St. Louis.

Each time she visits her relatives, she either flies or drives.

When travelling to Nashville, the probability that she drives is , and when travelling to St. Louis, the probability that she flies is .

Given that the probability that she drives when visiting her relatives is , find the probability that for a particular trip,

she travels to Nashville;

**19b.** *[3 marks]*

she is on her way to Nashville, given that she is flying.

**20a.** *[1 mark]*

Ava and Barry play a game with a bag containing one green marble and two red marbles. Each player in turn randomly selects a marble from the bag, notes its colour and replaces it. Ava wins the game if she selects a green marble. Barry wins the game if he selects a red marble. Ava starts the game.

Find the probability that Ava wins on her first turn.

**20b.** *[2 marks]*

Find the probability that Barry wins on his first turn.

**20c.** *[4 marks]*

Find the probability that Ava wins in one of her first three turns.

**20d.** *[4 marks]*

Find the probability that Ava eventually wins.

**21a.** *[4 marks]*

Find the term in  in the expansion of .

**21b.** *[4 marks]*

Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw

it to decide if they are going to eat a cookie.

Mina throws her die just once and she eats a cookie if she throws a four, a five or a six.

Norbert throws his die six times and each time eats a cookie if he throws a five or a six.

Calculate the probability that five cookies are eaten.

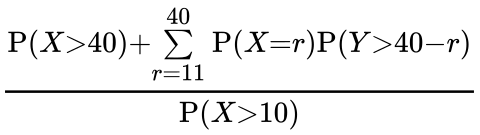
**22a.** *[2 marks]*

The number of birds seen on a power line on any day can be modelled by a Poisson distribution with mean 5.84.

Find the probability that during a certain seven-day week, more than 40 birds have been seen on the power line.

**22b.** *[5 marks]*

On Monday there were more than 10 birds seen on the power line. Show that the probability of there being more than 40 birds seen on the power line from that Monday to the following Sunday, inclusive, can be expressed as:

 where  and .

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